

Indian Statistical Institute, Bangalore Centre
B.Math. (I Year) : 2014-2015
Semester II : Backpaper Examination
Probability Theory II

July 2015

Time: 3 hours

Maximum Marks : 100

Note: State clearly the results you are using in your answers.

1. (15 marks) Suppose the times it takes two students to solve a problem are independently and exponentially distributed with parameter λ . Find the probability that the first student will take at least twice as long as the second student to solve the problem.
2. (12 + 13 = 25 marks) Let

$$f(x, y) = C \exp\left\{-\frac{1}{2}(x^2 - xy + 7y^2)\right\}, (x, y) \in \mathbb{R}^2.$$

- (i) Find C so that $f(\cdot, \cdot)$ is a probability density function on \mathbb{R}^2 .
 - (ii) Find the marginal probability density functions. Are they independent?
3. (15 marks) Let X, Y be independent real valued random variables each having a uniform distribution on $(0, 1)$. Find the probability density function of $X + Y$.
 4. (20 marks) Let $a, b, c, d \in \mathbb{R}$ such that $ad - bc \neq 0$, and let A be the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Let $X = (X_1, X_2)$ be a two dimensional random variable having bivariate normal distribution with means $\mu_1, \mu_2 \in \mathbb{R}$, with variances $\sigma_1^2, \sigma_2^2 \in (0, \infty)$, and correlation coefficient $\rho \in (-1, 1)$. Define $Y = (Y_1, Y_2)$ by $Y = AX$. Find the distribution of Y , and identify the means, variances and the correlation coefficient.

5. (7 + 8 = 15 marks) X_1, X_2, X_3 are independent standard normal random variables. Indicating clearly the results you are using, find the distributions of (i) $X_1^2 + X_2^2 + X_3^2$, (ii) $(X_1^2 + X_2^2)/2X_3^2$.

6. (10 marks) Let X have a uniform distribution on $(-1, 1)$. Find the characteristic function of X .